



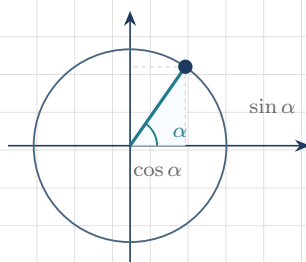
Trigonometry

Formula Reference

Trigonometry Formula Sheet · Reduction Formula Sheet · The Unit Circle Chart

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

HIGH SCHOOL PRECALCULUS



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Trigonometry Formula Sheet

√x ON THE UNIT CIRCLE

$$\cos \alpha = x_P \quad \sin \alpha = y_P \quad \tan \alpha = \frac{\sin \alpha}{\cos \alpha} \quad \cot \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad \tan \alpha \cdot \cot \alpha = 1$$

$$\tan^2 \alpha + 1 = \frac{1}{\cos^2 \alpha} \quad \cot^2 \alpha + 1 = \frac{1}{\sin^2 \alpha}$$

√x THE TABLE

	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	—
cot	—	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

√x SIGNS, PARITY AND PERIOD

	I	II	III	IV
sin	+	+	-	-
cos	+	-	-	+
tan, cot	+	-	+	-

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

$$\sin, \cos : \text{period } 2\pi \quad \tan, \cot : \text{period } \pi$$

√x ADDITION

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

√x DOUBLE AND HALF ANGLE

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha \quad \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1$$

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2} \quad \cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2}$$

$$\tan \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$$

√* SUMS AND PRODUCTS

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2} \quad \sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2} \quad \cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

√* THE UNIVERSAL SUBSTITUTION

With $t = \tan \frac{\alpha}{2}$:

$$\sin \alpha = \frac{2t}{1+t^2} \quad \cos \alpha = \frac{1-t^2}{1+t^2} \quad \tan \alpha = \frac{2t}{1-t^2}$$

√* GENERAL SOLUTIONS

$$\sin x = a \iff x = (-1)^k \arcsin a + k\pi, \quad |a| \leq 1$$

$$\cos x = a \iff x = \pm \arccos a + 2k\pi, \quad |a| \leq 1$$

$$\tan x = a \iff x = \arctan a + k\pi, \quad a \in \mathbb{R}$$

√* HOMOGENEOUS EQUATIONS

$$a \sin x + b \cos x = 0 \implies \tan x = -\frac{b}{a}$$

$$a \sin^2 x + b \sin x \cos x + c \cos^2 x = 0 \implies a \tan^2 x + b \tan x + c = 0$$

Divide by the highest power of $\cos x$ present. This is safe: $\sin x$ and $\cos x$ are never zero together.

√* SINE AND COSINE RULES

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2R$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha \quad \cos \alpha = \frac{b^2 + c^2 - a^2}{2bc}$$

$$P = \frac{1}{2}ab \sin \gamma$$

√* THE GENERAL WAVE

$$y = A \sin(\omega x + \varphi) + D$$

$$\text{amplitude } |A| \quad \text{period } T = \frac{2\pi}{|\omega|}$$

$$\text{horizontal shift } -\frac{\varphi}{\omega} \quad \text{vertical shift } D$$

Reduction Formula Sheet

√* THE TWO RULES — ALL YOU ACTUALLY NEED

Write the angle as $n \cdot 90^\circ \pm \alpha$ with α acute.

$$n \text{ odd} \implies \sin \leftrightarrow \cos, \quad \tan \leftrightarrow \cot$$

$$n \text{ even} \implies \text{the function is unchanged}$$

The sign is the one the **original** function carries in the quadrant where the **original** angle lands.

√* SECOND QUADRANT

$$\sin(90^\circ + \alpha) = \cos \alpha \quad \cos(90^\circ + \alpha) = -\sin \alpha$$

$$\tan(90^\circ + \alpha) = -\cot \alpha \quad \cot(90^\circ + \alpha) = -\tan \alpha$$

$$\sin(180^\circ - \alpha) = \sin \alpha \quad \cos(180^\circ - \alpha) = -\cos \alpha$$

$$\tan(180^\circ - \alpha) = -\tan \alpha \quad \cot(180^\circ - \alpha) = -\cot \alpha$$

√* THIRD QUADRANT

$$\sin(180^\circ + \alpha) = -\sin \alpha \quad \cos(180^\circ + \alpha) = -\cos \alpha$$

$$\tan(180^\circ + \alpha) = \tan \alpha \quad \cot(180^\circ + \alpha) = \cot \alpha$$

$$\sin(270^\circ - \alpha) = -\cos \alpha \quad \cos(270^\circ - \alpha) = -\sin \alpha$$

$$\tan(270^\circ - \alpha) = \cot \alpha \quad \cot(270^\circ - \alpha) = \tan \alpha$$

√* FOURTH QUADRANT

$$\sin(270^\circ + \alpha) = -\cos \alpha \quad \cos(270^\circ + \alpha) = \sin \alpha$$

$$\tan(270^\circ + \alpha) = -\cot \alpha \quad \cot(270^\circ + \alpha) = -\tan \alpha$$

$$\sin(360^\circ - \alpha) = -\sin \alpha \quad \cos(360^\circ - \alpha) = \cos \alpha$$

$$\tan(360^\circ - \alpha) = -\tan \alpha \quad \cot(360^\circ - \alpha) = -\cot \alpha$$

√* COMPLEMENTARY AND NEGATIVE ANGLES

$$\sin(90^\circ - \alpha) = \cos \alpha \quad \cos(90^\circ - \alpha) = \sin \alpha$$

$$\tan(90^\circ - \alpha) = \cot \alpha \quad \cot(90^\circ - \alpha) = \tan \alpha$$

$$\sin(-\alpha) = -\sin \alpha \quad \cos(-\alpha) = \cos \alpha \quad \tan(-\alpha) = -\tan \alpha$$

√* WHOLE TURNS

$$\sin(\alpha + 2k\pi) = \sin \alpha \quad \cos(\alpha + 2k\pi) = \cos \alpha$$

$$\tan(\alpha + k\pi) = \tan \alpha \quad \cot(\alpha + k\pi) = \cot \alpha$$

Remove whole turns **before** reducing: the formulas above assume an angle below 360° .

The Unit Circle Chart

